# Investigating texture six zero lepton mass matrices

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**Abstract**

All possible hermitian texture 6 zero Fritzsch like as well as non Fritzsch like lepton mass matrices (144 combinations in all) have been investigated for both Majorana and Dirac neutrinos for their compatibility with the current neutrino oscillation data, keeping in mind the hierarchy of neutrino masses. All the combinations considered here are ruled out by the existing data in the case of inverted hierarchy and degenerate scenario of Majorana neutrino masses. For Majorana neutrinos with normal hierarchy, we find that out of 144 combinations, only 16 possibilities can accommodate the experimental data. Assuming neutrinos to be Dirac particles, normal hierarchy, inverted hierarchy as well as degenerate neutrinos are ruled out for all the combinations.

## 1 Introduction

Evidence of neutrino oscillations, as obtained from solar neutrino experiments [1], atmospheric neutrino experiments [2], reactor based experiments [3] and accelerator based experiments [4], has firmly established the existence of massive neutrinos and mixing of lepton flavors. In the last few years, there has been considerable progress in the measurement of neutrino mass square differences and mixing angles, however, the absolute neutrino mass scale and the smallest lepton mixing angle $\theta_{13}$ are still unknown. Further, the presently available neutrino oscillation data does not throw any light on the neutrino mass hierarchy, which may be normal/inverted hierarchy and may even be degenerate. Furthermore, the situation becomes complicated when one realizes that neutrino masses are much smaller than charged fermion masses as well as it is yet not clear whether neutrinos are Dirac or Majorana particles. In the absence of a convincing fermion flavor theory in this regard, several approaches have been adopted, e.g., texture zeros [5], seesaw mechanism [6], radiative mechanisms [7], flavor symmetries [8], extra dimensions [9], etc..

In this context, texture specific mass matrices have received a good deal of attention in the literature, in particular, Fritzsch-like texture specific mass matrices seem to be very helpful in understanding the pattern of quark mixing and CP violation [10, 11]. Taking clues from the success of these matrices in the context of quarks, several attempts have been made to consider texture specific lepton mass matrices [12]-[16] for explaining the pattern of neutrino masses and mixing. In the absence of a sufficient amount of data regarding neutrino masses and mixing, it would require a very careful scrutiny of all possible textures to find viable structures, which are compatible with data and theoretical ideas, so that these are kept in mind while formulating mass matrices at the grand unified theory (GUT) scale.

In the quark sector, both Fritzsch like as well as non Fritzsch like texture six zero mass matrices have been completely ruled out [17]. In leptonic sector, most of the analysis have been carried out in the flavor basis [14], whereas in non flavor basis, when both charged lepton mass matrix $M_l$ and neutrino mass matrix $M_\nu$ are three zero type, the number of possibilities for texture six zero mass matrices becomes very large. These possibilities have been explored in the literature [15] both for Majorana as well as for Dirac neutrinos, however adequate attention has not been given to the cases for inverted hierarchy and degenerate scenarios. Also, it is perhaps desirable to note that Dirac neutrinos have not yet been ruled out by the experiments [18], therefore it becomes important in the case of texture six zero possibilities to carry out a detailed comparison for Dirac like as well as Majorana like neutrinos in the case of normal, inverted and degenerate cases. This exercise becomes all the more interesting in view of the refinements of data and advocacy of quark lepton symmetry [19].

The purpose of the present paper is to update the analysis of [15] for all possibilities of texture six zero lepton mass matrices as well as to extend this analysis to the case of inverted and degenerate neutrino masses. To preserve the parallelism between quarks and leptons only those neutrino mass matrices have been considered, which are consistent with the requirement of non zero and distinct neutrino masses. Following our analysis in the quark sector [17], the mass matrix for leptons and neutrinos are taken to be hermitian. For the sake of completion, we have also investigated the cases corresponding to charged leptons being in the flavor basis. It would also be a desirable exercise to calculate several phenomenological quantities, such as effective neutrino mass $\langle \mathrm{m_{ee}} \rangle$ related to neutrinoless double beta decay, Jarlskog's rephasing invariant parameter in the leptonic sector $\mathrm{J_l}$ and the corresponding Dirac like CP violating phase $\delta_\mathrm{l}$ for the viable cases.

The detailed plan of the paper is as follows. In section 2, we present the essentials of the formalism connecting the mass matrix to the neutrino mixing matrix. Inputs used in the present analysis have been given in section 3. In section 4, various possibilities of texture six zero mass matrices have been given. In sections 5 and 6, for Majorana and Dirac neutrinos respectively, the detailed calculations pertaining to normal/inverted and degenerate cases have been discussed. Finally, section 7, summarizes our conclusions.

## 2 Construction of PMNS matrix from mass matrices

To begin with, we present the Fritzsch like hermitian texture six zero lepton mass matrices, e.g.,

$$M_l = \begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & C_l \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & C_\nu \end{pmatrix}, \tag{1}$$

$M_l$ and $M_{\nu D}$ respectively corresponding to charged lepton mass matrix and Dirac-like neutrino mass matrix. It may be noted that each of the above matrix is texture three zero type with $A_{l(\nu)} = |A_{l(\nu)}|e^{i\alpha_{l(\nu)}}$ and $B_{l(\nu)} = |B_{l(\nu)}|e^{i\beta_{l(\nu)}}$. For Majorana neutrinos, the neutrino mass matrix $M_\nu$ is given by seesaw mechanism [6], for example,

$$M_\nu = -M_{\nu D}^T \, (M_R)^{-1} \, M_{\nu D}, \tag{2}$$

where $M_{\nu D}$ and $M_R$ are respectively, the Dirac neutrino mass matrix and the right-handed Majorana neutrino mass matrix. It may be mentioned that for both Majorana as well as Dirac neutrinos the texture is imposed only on $M_{\nu D}$, with no such restriction on $M_\nu$ for Majorana case. In the absence of any guidelines for the right handed Majorana mass matrix $M_R$ as well as to keep the number of parameters under control, it would be desirable to keep its structure as simple as possible, therefore we take $M_R = m_R I$ where $I$ is the unity matrix and $m_R$ denotes a very large mass scale.

To fix the notations and conventions as well as to facilitate the understanding of inverted hierarchy case and its relationship to normal hierarchy case, we detail the formalism connecting the mass matrices to the neutrino mixing matrix. To facilitate the diagonalization of $M_k$, where $k = l, \nu D$, the complex mass matrix $M_k$ can be expressed as

$$M_k = Q_k M_k^r P_k \tag{3}$$

or

$$M_k^r = Q_k^\dagger M_k P_k^\dagger \,, \tag{4}$$

where $M_k^r$ is a real symmetric matrix with real eigenvalues and $Q_k$ and $P_k$ are diagonal phase matrices. For the hermitian mass matrix $Q_k = P_k^\dagger$. In general, the real matrix $M_k^r$ is diagonalized by the orthogonal transformation $O_k$, e.g.,

$$M_k^{diag} = {O_k}^T M_k^r O_k \,, \tag{5}$$

which on using equation (4) can be rewritten as

$$M_k^{diag} = {O_k}^T Q_k^\dagger M_k P_k^\dagger O_k \,. \tag{6}$$

To facilitate the construction of diagonalization transformations for different hierarchies, we introduce a diagonal phase matrix $\xi_k$ defined as diag(1, $e^{i\pi}$, 1) for the case of normal hierarchy and as diag(1, $e^{i\pi}$, $e^{i\pi}$) for the case of inverted hierarchy. Equation (6) can now

be written as

$$\xi_k M_k^{diag} = {O_k}^T Q_k^\dagger M_k P_k^\dagger O_k \,, \tag{7}$$

which can also be expressed as

$$M_k^{diag} = \xi_k^\dagger {O_k}^T Q_k^\dagger M_k P_k^\dagger O_k \,. \tag{8}$$

Making use of the fact that $O_k^* = O_k$ it can be further expressed as

$$M_k^{diag} = (Q_k O_k \xi_k)^\dagger M_k (P_k^\dagger O_k), \tag{9}$$

from which one gets

$$M_k = Q_k O_k \xi_k M_k^{diag} O_k^T P_k. \tag{10}$$

The case of leptons is fairly straight forward, whereas in the case of neutrinos, the diagonalizing transformation is hierarchy specific as well as requires some fine tuning of the phases of the right handed neutrino mass matrix $M_R$. To clarify this point further, in analogy with equation (10), we can express $M_{\nu D}$ as

$$M_{\nu D} = Q_{\nu D} O_{\nu D} \xi_{\nu D} M_{\nu D}^{diag} O_{\nu D}^T P_{\nu D}. \tag{11}$$

Substituting the above value of $M_{\nu D}$ in equation (2) one obtains

$$M_\nu = -(Q_{\nu D} O_{\nu D} \xi_{\nu D} M_{\nu D}^{diag} O_{\nu D}^T P_{\nu D})^T (M_R)^{-1} (Q_{\nu D} O_{\nu D} \xi_{\nu D} M_{\nu D}^{diag} O_{\nu D}^T P_{\nu D}), \tag{12}$$

which, on using $P_{\nu D}^T = P_{\nu D}$, can further be written as

$$M_\nu = -P_{\nu D} O_{\nu D} M_{\nu D}^{diag} \xi_{\nu D} O_{\nu D}^T Q_{\nu D}^T (M_R)^{-1} Q_{\nu D} O_{\nu D} \xi_{\nu D} M_{\nu D}^{diag} O_{\nu D}^T P_{\nu D}, \tag{13}$$

wherein, assuming fine tuning, the phase matrices $Q_{\nu D}^T$ and $Q_{\nu D}$ along with $-M_R$ can be taken as $m_R$ diag$(1,1,1)$ as well as using the unitarity of $\xi_{\nu D}$ and orthogonality of $O_{\nu D}$, the above equation can be expressed as

$$M_\nu = P_{\nu D} O_{\nu D} \frac{(M_{\nu D}^{diag})^2}{m_R} O_{\nu D}^T P_{\nu D}. \tag{14}$$

The lepton mixing matrix or the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [20] $U$ can be obtained from the matrices used for diagonalizing the mass matrices $M_l$ and $M_\nu$ and is expressed as

$$U = (Q_l O_l \xi_l)^\dagger (P_{\nu D} O_{\nu D}). \tag{15}$$

Eliminating the phase matrix $\xi_l$ by redefinition of the charged lepton phases, the above equation becomes

$$U = O_l^\dagger Q_l P_{\nu D} O_{\nu D} \,, \tag{16}$$

where $Q_l P_{\nu D}$, without loss of generality, can be taken as diag$(e^{i\phi_1},\, e^{i\phi_2},\, e^{i\phi_3})$; $\phi_1$, $\phi_2$ and $\phi_3$ being related to the phases of mass matrices and can be treated as free parameters.

When $M_l$ and $M_\nu$ both are texture three zero type, it is sufficient to consider only two phases, i.e. $\phi_1$ and $\phi_2$ or $\phi_1$ and $\phi_3$ or $\phi_2$ and $\phi_3$.

# 3 Inputs used in the present analysis

Before going into the details of the analysis, we would like to mention some of the essentials pertaining to various inputs. The inputs for masses and mixing angles used in the present analysis at $3\sigma$ C.L. are as follows [21],

$$\Delta m_{12}^2 = (6.90 - 8.20) \times 10^{-5}\ \mathrm{eV}^2, \quad \Delta m_{23}^2 = (2.09 - 2.83) \times 10^{-3}\ \mathrm{eV}^2, \tag{17}$$

$$\theta_{12} = 31.7^\circ - 37.7^\circ, \quad \theta_{23} = 35.2^\circ - 53.7^\circ, \quad \theta_{13} \leq 12.7^\circ. \tag{18}$$

The above data reveals that at present not much is known about the hierarchy of neutrino masses as well as about their absolute values.

The masses and mixing angles, used in the analysis, have been constrained by the data given in equations (17) and (18). For the purpose of calculations, we have taken the lightest neutrino mass and the phases $\phi_1$ and $\phi_2$ as free parameters. The other two masses are constrained by $\Delta m_{12}^2 = m_{\nu_2}^2 - m_{\nu_1}^2$ and $\Delta m_{23}^2 = m_{\nu_3}^2 - m_{\nu_2}^2$ in the normal hierarchy case defined as $m_{\nu_1} < m_{\nu_2} \ll m_{\nu_3}$ and also valid for the degenerate case defined as $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3}$ and by $\Delta m_{23}^2 = m_{\nu_2}^2 - m_{\nu_3}^2$ in the inverted hierarchy case defined as $m_{\nu_3} \ll m_{\nu_1} < m_{\nu_2}$. It may be noted that lightest neutrino mass corresponds to $m_{\nu_1}$ for the normal hierarchy case and to $m_{\nu_3}$ for the inverted hierarchy case. The explored range of lightest neutrino mass is taken to be $0.0001\,\mathrm{eV} - 1.0\,\mathrm{eV}$ as our results remain unaffected even if the range is extended further. In the absence of any constraints on the phases $\phi_1$ and $\phi_2$ these have been given full variation from 0 to $2\pi$.

# 4 Texture six zero lepton mass matrices

To begin with, we enumerate the number of possibilities for the texture six zero lepton mass matrices. It is easy to check from equation (1) that there are 20 possible patterns of texture three zero hermitian mass matrices, which differ from each other with regard to the position of zeros in the structure of mass matrix. Texture six zero mass matrices are obtained when both $M_l$ and $M_{\nu D}$ are texture three zero type, implying that there will be 400 combinations of texture six zero lepton mass matrices. As mentioned earlier, in the present work, we have considered only those mass matrices which lead to non zero and distinct mass eigenvalues, therefore imposing the trace and determinant condition on the mass matrix, i.e. Trace $M_{l,\nu D} \neq 0$ and Det $M_{l,\nu D} \neq 0$, we are left with 12 patterns, classified into 2 distinct classes, depending upon the diagonalization equations these satisfy, as given in Table (1). Details of the diagonalization equations for these 12 mass matrices can be looked up in our earlier work [17]. Matrices $M_l$ and $M_{\nu D}$ each can correspond to any of the 12 patterns, therefore yielding 144 possible combinations of texture six zero lepton mass matrices which in principle can yield neutrino mixing matrix. These 144

| | Class I | Class II |
|---|---|---|
| a | $\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & Be^{-i\beta} & C \end{pmatrix}$ | $\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & D & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C \end{pmatrix}$ |
| b | $\begin{pmatrix} \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & C & Be^{i\beta} \\ Ae^{-i\alpha} & Be^{-i\beta} & \mathbf{0} \end{pmatrix}$ | $\begin{pmatrix} \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & C & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & D \end{pmatrix}$ |
| c | $\begin{pmatrix} \mathbf{0} & Ae^{i\alpha} & Be^{i\beta} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & C \end{pmatrix}$ | $\begin{pmatrix} D & Ae^{i\alpha} & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C \end{pmatrix}$ |
| d | $\begin{pmatrix} C & Be^{i\beta} & \mathbf{0} \\ Be^{-i\beta} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$ | $\begin{pmatrix} C & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & D & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$ |
| e | $\begin{pmatrix} \mathbf{0} & Be^{i\beta} & Ae^{i\alpha} \\ Be^{-i\beta} & C & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \end{pmatrix}$ | $\begin{pmatrix} D & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & C & \mathbf{0} \\ Ae^{-i\alpha} & \mathbf{0} & \mathbf{0} \end{pmatrix}$ |
| f | $\begin{pmatrix} C & \mathbf{0} & Be^{i\beta} \\ \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ Be^{-i\beta} & Ae^{-i\alpha} & \mathbf{0} \end{pmatrix}$ | $\begin{pmatrix} C & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & Ae^{i\alpha} \\ \mathbf{0} & Ae^{-i\alpha} & D \end{pmatrix}$ |

Table 1: Table showing various patterns of texture three zero mass matrices categorized into two classes, class I and II.

combinations further form 4 different categories e.g.;
Category 1: $M_l$ from class I and $M_{\nu D}$ from class I.
Category 2: $M_l$ from class II and $M_{\nu D}$ from class II.
Category 3: $M_l$ from class I and $M_{\nu D}$ from class II.
Category 4: $M_l$ from class II and $M_{\nu D}$ from class I.

Each category corresponds to 36 possibilities which need exhaustive analysis. As mentioned earlier, we have considered the cases of normally hierarchical, inversely hierarchical and degenerate Majorana as well as Dirac neutrinos. Thus, for each of 144 combinations, there are 6 cases which have to analyzed leading to a total of 864 possibilities for texture six zero lepton mass matrices.

# 5 Majorana Neutrinos

To begin with, we have analyzed 144 combinations of texture six zero lepton mass matrices corresponding to normal hierarchy of Majorana neutrinos, by confronting their corresponding mixing matrix against latest neutrino oscillation data given in section (3). It may be mentioned again that for Majorana neutrinos, the hermitian texture three zero structure is imposed on $M_{\nu D}$ as given in equation (2).

In literature, the details of Fritzsch like texture six zero lepton mass matrices [12]-[16] have been presented by several authors, however, for non-Fritzsch like mass matrices similar details have not been presented. In this context, to detail the methodology, we consider a typical combination corresponding to category 4, where both $M_l$ and $M_{\nu D}$ are non Fritzsch type, for example,

$$M_l = \begin{pmatrix} C_l & 0 & 0 \\ 0 & D_l & A_l \\ 0 & A_l^* & 0 \end{pmatrix}, \qquad M_{\nu D} = \begin{pmatrix} 0 & 0 & A_\nu \\ 0 & C_\nu & B_\nu \\ A_\nu^* & B_\nu^* & 0 \end{pmatrix}. \tag{19}$$

The diagonalization transformations for these matrices can be easily obtained in terms of neutrino masses $m_{\nu_1}$, $m_{\nu_2}$ and $m_{\nu_3}$ for both normal and inverted hierarchy and are given in the Appendix A. The phase matrix $Q_l P_{\nu D}$ for this particular combination is given as diag($e^{i\phi_1}$, $e^{i\phi_2}$, 1). The scanned ranges of lightest neutrino mass $m_{\nu_1}$ and phases $\phi_1$ and $\phi_2$ have been given in section (3) and $\Delta m^2_{12}$, $\Delta m^2_{23}$ and the mixing angles have been constrained as given by equations (17) and (18). Using equation (16), the PMNS matrix, $U$, obtained for this particular combination is given as

$$U = \begin{pmatrix} 0.78-0.84 & 0.52-0.61 & 0.12-0.16 \\ 0.39-0.49 & 0.40-0.53 & 0.72-0.79 \\ 0.29-0.43 & 0.67-0.72 & 0.59-0.68 \end{pmatrix}. \tag{20}$$

On comparing equation (20) with the ranges of mixing matrix element given by Garcia et al. [22] at $3\sigma$ C.L., for example,

$$U = \begin{pmatrix} 0.79-0.86 & 0.50-0.61 & 0.00-0.20 \\ 0.25-0.53 & 0.47-0.73 & 0.56-0.79 \\ 0.21-0.51 & 0.42-0.69 & 0.61-0.83 \end{pmatrix}, \tag{21}$$

one finds that the two have good overlap. To graphically show the viability of this particular combination of texture six zero mass matrices, in Figure (1) we have plotted the lightest neutrino mass against the mixing angle $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ by giving full variation to input parameters. The dashed lines depict the limits obtained by assuming normal hierarchy and the solid horizontal lines show the experimental $3\sigma$ limits of the plotted mixing angles. A general look at the Figure (1), shows that mixing angles are well within their experimental ranges for a common neutrino mass range for normal hierarchy.

In a similar manner, one can check the viability of the above set of mass matrices

for inversely hierarchical Majorana neutrinos. In the same Figure (1), using dot-dashed lines, we have plotted the limits of mixing angles obtained by assuming inverted hierarchy against the lightest neutrino mass. It is immediately clear from Figures (1a) and (1c) that the obtained ranges of $\theta_{12}$ and $\theta_{23}$ are far from their experimental limits, thus ruling out the inverted hierarchy for this particular combination.

Coming to the degenerate scenarios of Majorana neutrinos characterized by either $m_{\nu_1} \lesssim m_{\nu_2} \sim m_{\nu_3} \sim 0.1$ eV or $m_{\nu_3} \sim m_{\nu_1} \lesssim m_{\nu_2} \sim 0.1$ eV corresponding to normal hierarchy and inverted hierarchy respectively, one can easily infer from Figures (1a) and (1c) that degenerate scenario is also ruled out. This can be understood by noting that around 0.1eV the limits obtained by assuming normal hierarchy and inverted hierarchy have no overlap with experimental limits of $\theta_{12}$ and $\theta_{23}$.

The viability of rest of the 143 combinations can similarly be checked for normal hierarchy, inverted hierarchy and for degenerate Majorana neutrinos. For normal hierarchy, we find that in category 1, there are 12 viable combinations. Numerical results corresponding to one such parallel combination $\mathrm{I}_a\mathrm{I}_a$ are given in the first row of Table (2). The spectrum of neutrino masses shows that neutrinos are following normal hierarchy. Further, $\theta_{12}$ and $\theta_{13}$ are spanning their full experimental range, however the obtained range of $\theta_{23}$ is just below its maximal value. One finds that $\theta_{23}$ is sensitive to variations in the mass squared differences, however it is not possible to obtain a higher value for $\theta_{23}$ even when the ranges of $\Delta m_{12}^2$ and $\Delta m_{23}^2$ are extended further.

Apart from mixing angles, we have also calculated the Jarlskog's rephasing invariant in the leptonic sector $J_l$, Dirac like CP violating phase $\delta_l$ and effective neutrino mass $\langle m_{ee}\rangle$ related to neutrinoless double beta decay $\beta\beta_{0\nu}$. The parameter $J_l$ has been calculated by using the expression

$$J_l = \mathrm{Im}[U_{23}U_{33}^*U_{22}U_{32}^*], \tag{22}$$

where $U_{23}$, $U_{33}$, $U_{22}$ and $U_{32}$ are the elements of mixing matrix U given in equation (16).

The Dirac like CP violating phase $\delta_l$ can be determined from

$$J_l = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2\sin\delta_l, \tag{23}$$

where $s_{12}$, $s_{13}$ and $s_{23}$ are sines of mixing angles $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$.

The effective Majorana mass to be measured in $\beta\beta_{0\nu}$ decay experiment, is given as

$$\langle m_{ee}\rangle = m_{\nu1}U_{11}^2 + m_{\nu2}U_{12}^2 + m_{\nu3}U_{13}^2. \tag{24}$$

The obtained ranges of $J_l$, $\delta_l$ and $\langle m_{ee}\rangle$ are in agreement with the ranges obtained in other such analysis [16, 22]. It may be mentioned that the other parallel combinations e.g. $\mathrm{I}_b\mathrm{I}_b$, $\mathrm{I}_c\mathrm{I}_c$, $\mathrm{I}_d\mathrm{I}_d$, $\mathrm{I}_e\mathrm{I}_e$ and $\mathrm{I}_f\mathrm{I}_f$ are isomeric to $\mathrm{I}_a\mathrm{I}_a$, i.e., these combinations are although structurally different from each other, their predictions for lepton masses, mixing matrix elements and for other phenomenological quantities mentioned above are exactly same and hence are not given in the table. Similarly, category 1 also has 6 non parallel viable combinations such as $\mathrm{I}_a\mathrm{I}_b$, isomeric to combinations $\mathrm{I}_b\mathrm{I}_a$, $\mathrm{I}_c\mathrm{I}_f$, $\mathrm{I}_f\mathrm{I}_c$, $\mathrm{I}_d\mathrm{I}_e$, $\mathrm{I}_e\mathrm{I}_d$, given in the second row of the Table (2). It is clear from the table that the results for the non

| $\mathbf{M_l}$ $\mathbf{M_{\nu D}}$ | Neutrino masses | Mixing angles | $\langle \mathbf{m_{ee}} \rangle$ | $\mathbf{J_l}$ | $\delta_l$ |
|---|---|---|---|---|---|
| $\begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & C \end{pmatrix} \begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu^* & 0 & B_\nu \\ 0 & B_\nu^* & C \end{pmatrix}$ | $m_{\nu_1} = 0.0005 - 0.0044$ <br> $m_{\nu_2} = 0.0083 - 0.0100$ <br> $m_{\nu_3} = 0.0464 - 0.0541$ | $\theta_{12} = 31^o - 38^o$ <br> $\theta_{13} = 3^o - 11^o$ <br> $\theta_{23} = 35^o - 45^o$ | 0.0029-0.0081 | 0.0-0.024 | $0 - 49^o$ |
| $\begin{pmatrix} 0 & A_l & 0 \\ A_l^* & 0 & B_l \\ 0 & B_l^* & C_l \end{pmatrix} \begin{pmatrix} 0 & 0 & A_\nu \\ 0 & C_\nu & B_\nu \\ A_\nu^* & B_\nu^* & 0 \end{pmatrix}$ | $m_{\nu_1} = 0.0005 - 0.0038$ <br> $m_{\nu_2} = 0.0083 - 0.0098$ <br> $m_{\nu_3} = 0.0464 - 0.0540$ | $\theta_{12} = 31^o - 38^o$ <br> $\theta_{13} = 4^o - 12^o$ <br> $\theta_{23} = 45^o - 54^o$ | 0.0034 -0.0065 | 0.0-0.025 | $0 - 46^o$ |
| $\begin{pmatrix} C_l & 0 & 0 \\ 0 & 0 & A_l \\ 0 & A_l^* & D_l \end{pmatrix} \begin{pmatrix} 0 & 0 & A_\nu \\ 0 & C_\nu & B_\nu \\ A_\nu^* & B_\nu^* & 0 \end{pmatrix}$ | $m_{\nu_1} = 0.0008 - 0.0024$ <br> $m_{\nu_2} = 0.0083 - 0.0094$ <br> $m_{\nu_3} = 0.0465 - 0.0540$ | $\theta_{12} = 31^o - 38^o$ <br> $\theta_{13} = 7^o - 9^o$ <br> $\theta_{23} = 35^o - 44^o$ | 0.0037 -0.0061 | 0.0-0.015 | $0 - 27^o$ |
| $\begin{pmatrix} C_l & 0 & 0 \\ 0 & D_l & A_l \\ 0 & A_l^* & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & A_\nu \\ 0 & C_\nu & B_\nu \\ A_\nu^* & B_\nu^* & 0 \end{pmatrix}$ | $m_{\nu_1} = 0.0008 - 0.0025$ <br> $m_{\nu_2} = 0.0083 - 0.0094$ <br> $m_{\nu_3} = 0.0460 - 0.0540$ | $\theta_{12} = 31^o - 38^o$ <br> $\theta_{13} = 7^o - 9^o$ <br> $\theta_{23} = 46^o - 54^o$ | 0.0037 -0.0061 | 0.0-0.014 | $0 - 24^o$ |

Table 2: Calculated ranges of neutrino masses, mixing angles, $\langle \mathrm{m_{ee}} \rangle$, $\mathrm{J_l}$ and $\delta_\mathrm{l}$ for viable combinations of $M_l$ and $M_{\nu D}$ for normally hierarchical Majorana neutrinos.

parallel combinations are similar to the parallel combinations except that the obtained range of $\theta_{23}$ is above its maximal value. Further, categories 2 and 3 do not have any viable combination, because of only two generation mixing in the neutrino mass matrix. Lastly, category 4 does have four viable combinations such as $\mathrm{II}_f\mathrm{I}_b$ isomeric to $\mathrm{II}_d\mathrm{I}_a$ and $\mathrm{II}_d\mathrm{I}_b$ isomeric to $\mathrm{II}_f\mathrm{I}_a$, given respectively in the third and fourth row of the Table (2). The above two sets can be distinguished again on the basis of $\theta_{23}$, as combination $\mathrm{II}_f\mathrm{I}_b$ gives $\theta_{23}$ below the maximal value while $\mathrm{II}_d\mathrm{I}_b$ gives $\theta_{23}$ above its maximal value. All the four combinations of category 4 lead to a very constrained range of $\theta_{13}$ e.g. $7^\circ - 9^\circ$. It may be noted that although most of the phenomenological implications of the above mentioned 16 texture six zero lepton mass matrices are similar, however still these can be experimentally distinguished with more precise measurements of $\theta_{23}$ and $\theta_{13}$. The above mentioned texture combinations are found to be compatible with the current data even when the inputs are at their $2\sigma$ C.L..

In the case of inverted hierarchy and degenerate scenario of Majorana neutrinos, a similar analysis has been carried out for all the 144 combinations to check for their compatibility with the latest data. Interestingly, we find that inverted hierarchy as well as degenerate Majorana neutrinos are completely ruled out for hermitian texture six zero lepton mass matrices.

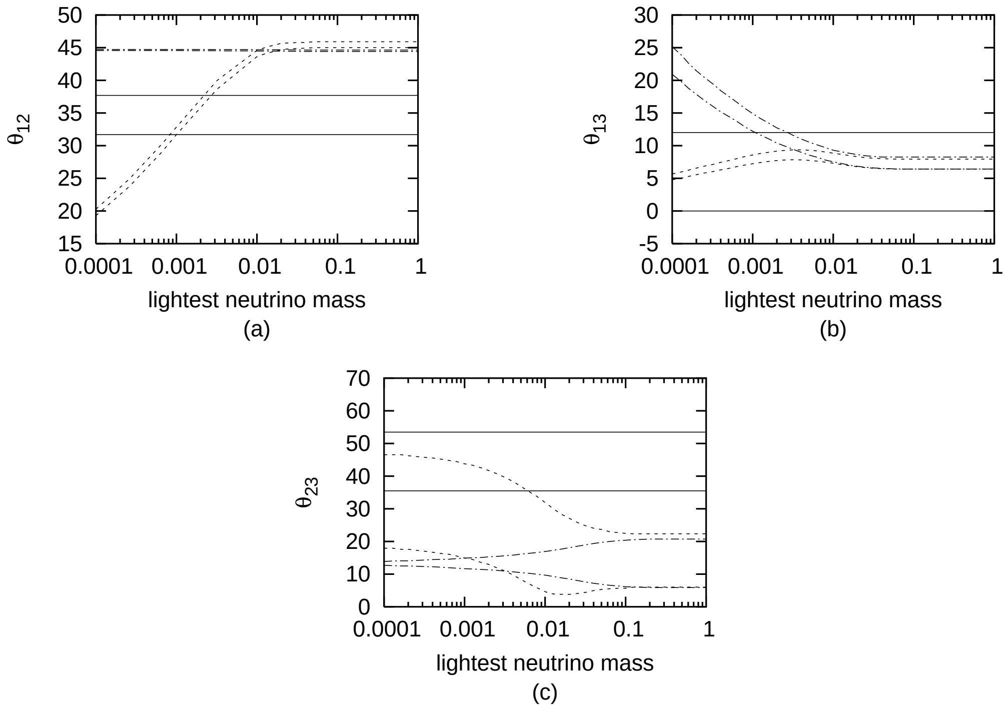


Figure 1: Plots showing variation of mixing angles $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ with lightest neutrino mass for $\mathrm{II_d I_b}$ combination given in equation (19) for Majorana neutrinos. The dashed lines and the dot-dashed lines depict the limits obtained assuming normal and inverted hierarchy respectively, the solid horizontal lines show the experimental $3\sigma$ limits.

# 6 Dirac Neutrinos

Coming to the case of Dirac neutrinos, we have again analyzed 144 combinations for normal hierarchy, inverted hierarchy and degenerate neutrinos. To illustrate the comparison with the Majorana neutrino case, we pick up the same non-Fritzsch like combination given in equation (19), to check its compatibility with the latest neutrino mixing data. The diagonalization transformations for these matrices can easily be obtained in terms of neutrino masses $m_{\nu_1}$, $m_{\nu_2}$ and $m_{\nu_3}$ for both normal and inverted hierarchy and are given in the Appendix A. The phase matrix $Q_l P_{\nu D}$ and the scanned ranges of lightest neutrino mass and phases $\phi_1$ and $\phi_2$ have already been mentioned above.

To check the compatibility of this particular combination for normal hierarchy of Dirac neutrinos, in Figure (2) we have plotted the allowed parameter space for $\theta_{12}$ and $\theta_{23}$ in $m_{\nu_1}$-$\theta_{13}$ plane, represented respectively by dots and crosses. One can immediately find from the figure that there is no common parameter space available to $\theta_{12}$ and $\theta_{23}$, concluding that this particular combination is not viable for Dirac neutrinos. This result remains unaffected even if the input parameter ranges are extended further. Also one can easily

check that mixing angle $\theta_{13}$ is coming out to be very small. Thus, normally hierarchical Dirac neutrinos are ruled out for the texture combination given in equation (19). Similarly, one can also show that degenerate and inversely hierarchical Dirac neutrinos for this particular combination are also ruled out.

The combinations which are viable for Majorana neutrinos, are not viable for Dirac neutrinos primarily because of mixing angle $\theta_{23}$. For example, for parallel combinations given in the first row as well as the combinations given in the third row of the Table (2), $\theta_{23}$ for Dirac neutrinos comes out to be below the experimental limits. Similarly, for non parallel combinations given in the second row and the combinations given in the fourth row of the Table (2), $\theta_{23}$ lies above its experimental limits.

A similar analysis carried out for the rest of the 128 combinations shows that there are no viable texture six zero lepton mass matrices for normally hierarchical, inversely hierarchical as well as for degenerate Dirac neutrinos, thus ruling out Dirac neutrinos completely for texture six zero mass matrices.

For the sake of completion, we have also analyzed the cases corresponding to charged leptons being in the flavor basis for Dirac as well as Majorana neutrinos and one finds that none of matrices gives result within the experimental ranges.

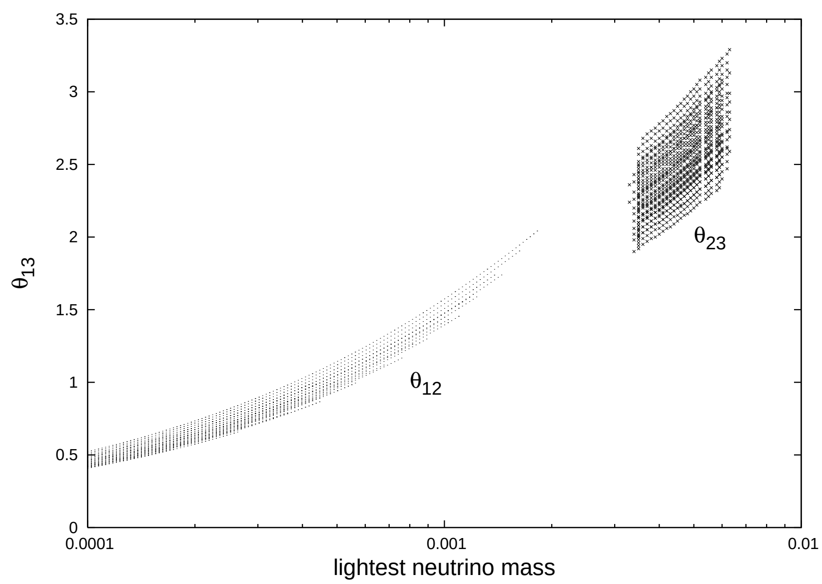


Figure 2: Allowed parameter space of $\theta_{12}$ and $\theta_{23}$ in 'lightest neutrino mass - $\theta_{13}$ plane' for normal hierarchy of Dirac neutrinos for texture six zero combination given in equation (19). The dots and crosses represent the allowed parameter space for $\theta_{12}$ and $\theta_{23}$ respectively.

# 7 Summary and Conclusions

To summarize, we have analyzed 144 possibilities corresponding to hermitian texture six zero lepton mass matrices to check for their viability against current neutrino oscillation data. For each of these mass matrices, various cases have been considered in the analysis, for example, normal hierarchy, inverted hierarchy and degenerate neutrinos for both Majorana as well as Dirac neutrinos. For Majorana neutrinos with normal hierarchy, out of 144, only 16 combinations are compatible with current neutrino oscillation data at $3\sigma$ C.L.. The above mentioned texture combinations are found to be compatible with current data even at $2\sigma$ level. The 16 viable combinations can be grouped in four sets such that the matrices placed in each set are isomeric to each other with regards to their predictions for lepton masses and flavor mixing parameters. It is important to note that these four sets can be experimentally distinguished from each other with more precise measurements of $\theta_{23}$ and $\theta_{13}$. The ranges of neutrino masses, the PMNS matrix, Jarlskog's rephasing parameter $\mathrm{J_l}$, Dirac like CP violating phase $\delta_\mathrm{l}$ and effective neutrino mass $\langle \mathrm{m_{ee}} \rangle$, calculated for viable combinations are in agreement with the ranges obtained in other such analysis. We find that inverted hierarchy and degenerate neutrinos are completely ruled out for texture six zero Majorana neutrinos. Interestingly, for Dirac neutrinos none of the 144 combinations is viable for normal hierarchy, inverted hierarchy as well as degenerate neutrinos, thus ruling out Dirac neutrinos completely for hermitian texture six zero lepton mass matrices.

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## Appendix A

The diagonalization transformations for the real texture 3 zero mass matrices given in equation (19) are as follows:

1. The diagonalizing matrix $O_l$ for the real 3 zero mass matrix $M_l$,

$$M_l = \begin{pmatrix} C_l & 0 & 0 \\ 0 & D_l & A_l \\ 0 & A_l^* & 0 \end{pmatrix}, \tag{25}$$

   is given as

$$O_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{\frac{m_\mu}{m_\mu+m_\tau}} & \sqrt{\frac{m_\tau}{m_\mu+m_\tau}} \\ 0 & -\sqrt{\frac{m_\tau}{m_\mu+m_\tau}} & \sqrt{\frac{m_\mu}{m_\mu+m_\tau}} \end{pmatrix}. \tag{26}$$

2. The diagonalizing matrix $O_{\nu D}$ for the real 3 zero neutrino mass matrix $M_{\nu D}$,

$$M_{\nu D} = \begin{pmatrix} 0 & 0 & A_\nu \\ 0 & C_\nu & B_\nu \\ A_\nu^* & B_\nu^* & 0 \end{pmatrix}, \tag{27}$$

   for normal hierarchy of neutrinos is given as

$$O_{\nu D} = \begin{pmatrix} -\sqrt{\frac{m_2 m_3(m_3-m_2)}{(m_1-m_2+m_3)(m_1+m_2)(m_3-m_1)}} & \sqrt{\frac{m_1 m_3(m_1+m_3)}{(m_1-m_2+m_3)(m_1+m_2)(m_3+m_2)}} & \sqrt{\frac{m_2 m_1(m_2-m_1)}{(m_1-m_2+m_3)(m_3+m_2)(m_3-m_1)}} \\ \sqrt{\frac{m_1(m_1+m_3)(m_2-m_1)}{(m_1-m_2+m_3)(m_1+m_2)(m_3-m_1)}} & \sqrt{\frac{m_2(m_2-m_1)(m_3-m_2)}{(m_1-m_2+m_3)(m_1+m_2)(m_2+m_3)}} & \sqrt{\frac{m_3(m_1+m_3)(m_3-m_2)}{(m_1-m_2+m_3)(m_3+m_2)(m_3-m_1)}} \\ \sqrt{\frac{m_1(m_3-m_2)}{(m_1+m_2)(m_3-m_1)}} & \sqrt{\frac{m_2(m_1+m_3)}{(m_1+m_2)(m_3+m_2)}} & \sqrt{\frac{m_3(m_2-m_1)}{(m_3-m_1)(m_3+m_2)}} \end{pmatrix}.$$

Similarly, for inverted hierarchy of neutrinos, $O_{\nu D}$ is given as

$$O_{\nu D} = \begin{pmatrix} \sqrt{\frac{m_2 m_3(m_3+m_2)}{(-m_1+m_2+m_3)(m_1+m_2)(m_3+m_1)}} & -\sqrt{\frac{m_1 m_3(m_1-m_3)}{(-m_1+m_2+m_3)(m_1+m_2)(m_3+m_2)}} & \sqrt{\frac{m_2 m_1(m_1-m_2)}{(-m_1+m_2+m_3)(m_3+m_1)(m_3-m_2)}} \\ -\sqrt{\frac{m_1(m_1-m_3)(m_2-m_1)}{(-m_1+m_2+m_3)(m_1+m_2)(m_3-m_1)}} & \sqrt{\frac{m_2(m_2-m_1)(m_3+m_2)}{(-m_1+m_2+m_3)(m_1+m_2)(m_2-m_3)}} & -\sqrt{\frac{m_3(m_2+m_3)(m_3-m_1)}{(-m_1+m_2+m_3)(m_3-m_2)(m_3+m_1)}} \\ \sqrt{\frac{m_1(m_3+m_2)}{(m_1+m_2)(m_3+m_1)}} & \sqrt{\frac{m_2(m_1-m_3)}{(m_1+m_2)(m_2-m_3)}} & \sqrt{\frac{m_3(m_1-m_2)}{(m_3-m_2)(m_3+m_1)}} \end{pmatrix},$$

where $m_1 = m_{\nu_1}$, $m_2 = m_{\nu_2}$ and $m_3 = m_{\nu_3}$ for Dirac neutrinos and $m_1 = \sqrt{m_{\nu_1} m_R}$, $m_2 = \sqrt{m_{\nu_2} m_R}$ and $m_3 = \sqrt{m_{\nu_3} m_R}$ for Majorana neutrinos.